Low-scale SUSY breaking and the (s)goldstino physics.

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Abstract

For a 4D N=1 supersymmetric model with a low SUSY breaking scale (f) and general Kahler potential $K(\Phi^i, \Phi_j^\dagger)$ and superpotential $W(\Phi^i)$ we study, in an effective theory approach, the relation of the goldstino superfield to the (Ferrara-Zumino) superconformal symmetry breaking chiral superfield X. In the presence of more sources of supersymmetry breaking, we verify the conjecture that the goldstino superfield is the (infrared) limit of X for zero-momentum and $\Lambda \to \infty$ (Λ is the effective cut-off scale). We then study the constraint $X^2 = 0$, which in the one-field case is known to decouple a massive sgoldstino and thus provide an effective superfield description of the Akulov-Volkov action for the goldstino. In the presence of additional fields that contribute to SUSY breaking we identify conditions for which $X^2 = 0$ remains valid, in the effective theory below a large but finite sgoldstino mass. The conditions ensure that the effective expansion (in $1/\Lambda$) of the initial Lagrangian is not in conflict with the decoupling limit of the sgoldstino ($1/m_{\text{sgoldstino}} \sim \Lambda/f$, $f < \Lambda^2$).

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1   Introduction

Supersymmetry, if realised in Nature, must be broken at some high scale. In this work we consider the case of SUSY breaking at a (low) scale $\sqrt{f} \ll M_{Planck}$ in the hidden sector, an example of which is gauge mediation. In this case the transverse gravitino couplings ($\sim 1/M_{Planck}$) can be neglected relative to their longitudinal counterparts ($\sim 1/\sqrt{f}$) that are due to its goldstino component. If so, one can then work in the gravity decoupled limit, with a massless goldstino. The auxiliary field of the goldstino superfield breaks SUSY spontaneously, while the goldstino scalar superpartner (sgoldstino) can acquire mass and decouple at low energy, similar to SM superpartners, to leave a non-linear SUSY realisation. To describe this regime one can work with component fields and integrate out explicitly the sgoldstino and other superpartners too (if massive), to obtain the effective Lagrangian. Alternatively one can use a less known but elegant superfield formalism endowed with constraints, see [1] for a review. If applied to matter, gauge and goldstino superfields, these constraints project out the massive superpartners, giving a superfield action for the *light* states. Such constraint may be applied only to the goldstino superfield which can be coupled to the linear multiplets of the model (e.g. MSSM), to parametrize SUSY breaking.

In this work we study SUSY breaking in the hidden sector and its relation to the goldstino superfield in the presence of more sources of SUSY breaking and the connection of the goldstino superfield to the superconformal symmetry breaking chiral superfield X [2].

It was noticed long ago [3] (see also [1]) that a Lagrangian, function of $\Phi^1 = (\phi^1, \psi^1, F^1)$

$$\mathcal{L} = \int d^4\theta \, \Phi_1^\dagger \, \Phi^1 + \left\{ \int d^2\theta \, f \, \Phi^1 + h.c. \right\}, \quad \text{with a constraint:} \quad (\Phi^1)^2 = 0, \qquad (1)$$

provides an onshell superfield description of the Akulov-Volkov action for the goldstino field [4]. Indeed, the constraint (which generates interactions) has a solution $\Phi^1 = \psi^1\psi^1/(2F^1) + \sqrt{2}\theta\psi^1 + \theta\theta F^1$, which "projects out" the sgoldstino ϕ^1. When this Φ^1 is used back in (1), one obtains the onshell-SUSY Lagrangian for a massless goldstino ψ^1. If more fields are present and for general K, W, the situation is more complicated and was little studied.

Further, it was only recently conjectured [1] that the goldstino superfield is the infrared (i.e. zero-momentum) limit of the superconformal symmetry breaking chiral superfield X, that breaks the conservation of the Ferrara-Zumino current [2]. In the light of the above discussion, one then also expects that in such limit $X^2 \sim (\Phi^1)^2 = 0$, and this conjecture was verified in very simple examples. We address these two problems, more exactly:

a) the convergence of the field X to the goldstino superfield, in the limit of vanishing momentum. We note that one must also take $\Lambda \to \infty$.

b) the validity and implications of the constraint $X^2 = 0$ [1, 3]. This is supposed to decouple (project out) the sgoldstino. In particular this can mean an infinite sgoldstino mass [5], however on dimensional grounds this is actually proportional to f/Λ. It is difficult to satisfy a) and b) simultaneously in general cases, because of opposite limits, of large

Λ and large sgoldstino mass, while $f < \Lambda^2$. This situation is further complicated by the presence of more fields, some of which can also contribute to (spontaneous) SUSY breaking.

The above problems were studied for simple superpotentials, like linear superpotentials [1, 6] or with only one field breaking SUSY [5], with many phenomenological applications studied in [7]. We investigate below problems a), b) for general K and W, with more fields contributing to SUSY breaking in the hidden sector. This helps a better understanding of SUSY breaking and its transmission to the visible sector via the coupling [1] of the X field to models like the Minimal Supersymmetric Standard Model (MSSM).

2 (s)Goldstino and its relation to the chiral superfield X.

2.1 Goldstino and sgoldstino eigenstates for arbitrary K, W.

The starting point is the general Lagrangian

$$
\begin{aligned}
L &= \int d^4\theta \, K(\Phi^i, \Phi_j^\dagger) + \left\{ \int d^2\theta \, W(\Phi^i) + \int d^2\overline{\theta} \, W^\dagger(\Phi_i^\dagger) \right\} \\
&= K_i{}^j \left[\partial_\mu \phi^i \, \partial^\mu \phi_j^\dagger + \frac{i}{2} \left(\psi^i \sigma^\mu \mathcal{D}_\mu \overline{\psi}_j - \mathcal{D}_\mu \psi^i \sigma^\mu \overline{\psi}_j \right) + F^i F_j^\dagger \right] \\
&\quad + \frac{1}{4} K_{ij}^{kl} \, \psi^i \psi^j \, \overline{\psi}_k \overline{\psi}_l + \left[\left(W_k - \frac{1}{2} K_k^{ij} \overline{\psi}_i \overline{\psi}_j \right) F^k - \frac{1}{2} W_{ij} \, \psi^i \psi^j + h.c. \right] \quad (2)
\end{aligned}
$$

where we ignored a $(-1/4)\Box K$ in the rhs. Here $K_i \equiv \partial K/\partial \phi^i$, $K^n \equiv \partial K/\partial \phi_n^\dagger$, $K_i^n \equiv \partial^2 K/(\partial \phi^i \, \partial \phi_n^\dagger)$, $W_j = \partial W/\partial \phi^j$, $W^j = (W_j)^\dagger$, etc, with $W = W(\phi^i)$, $K = K(\phi^i, \phi_j^\dagger)$.

Terms with more than two derivatives of K are suppressed by powers of Λ which is the UV cutoff of the model, $K_{ij}^k \sim 1/\Lambda$, $K_{km}^{ij} \sim 1/\Lambda^2$, etc. We also used the notation

$$
\begin{aligned}
\mathcal{D}_\mu \psi^l &\equiv \partial_\mu \psi^l - \Gamma_{jk}^l \, (\partial_\mu \phi^j) \, \psi^k, \qquad \Gamma_{jk}^l = (K^{-1})_m^l \, K_{jk}^m \\
\mathcal{D}_\mu \overline{\psi}_l &\equiv \partial_\mu \overline{\psi}_l - \Gamma_l^{jk} \, (\partial_\mu \phi_j^\dagger) \, \overline{\psi}_k, \qquad \Gamma_l^{jk} = (K^{-1})_l^m \, K_m^{jk}
\end{aligned} \quad (3)
$$

Eq.(2) is the offshell form of the Lagrangian. The eqs of motion for auxiliary fields

$$
\begin{aligned}
F_m^\dagger &= -(K^{-1})_m^i \, W_i + (1/2) \, \Gamma_m^{lj} \, \overline{\psi}_l \overline{\psi}_j \\
F^m &= -(K^{-1})_i^m \, W^i + (1/2) \, \Gamma_{lj}^m \, \psi^l \psi^j
\end{aligned} \quad (4)
$$

can be used to obtain the onshell form of L:

$$
\begin{aligned}
L &= K_i^j \left[\partial_\mu \phi^i \partial^\mu \phi_j^\dagger + \frac{i}{2} \left(\psi^i \sigma^\mu \, \mathcal{D}_\mu \overline{\psi}_j - \mathcal{D}_\mu \psi^i \sigma^\mu \overline{\psi}_j \right) \right] - W^k \, (K^{-1})_k^i \, W_i \\
&\quad - \frac{1}{2} \left[\left(W_{ij} - \Gamma_{ij}^m W_m \right) \psi^i \psi^j + h.c. \right] + \frac{1}{4} R_{ij}^{kl} \, \psi^i \psi^j \, \overline{\psi}_k \overline{\psi}_l, \qquad R_{ij}^{kl} = K_{ij}^{kl} - K_{ij}^n \Gamma_n^{kl} \quad (5)
\end{aligned}
$$

2

Here R^{kl}_{ij} is the curvature tensor and the potential of the model is

$$V = W_i \, (K^{-1})^i_j \, W^j \tag{6}$$

The derivatives of K, W are scalar fields-dependent. In the following we always work in normal coordinates, in which case $k^i_j = \delta^j_i$, $k^i_{jk...} = k^{jk...}_i = 0$, where $k^{i...}_{j...}$ are the values of $K^{i...}_{j...}$ evaluated on the ground state (denoted $\langle \phi^k \rangle$, $\langle F^k \rangle$, $\langle \psi^k \rangle = 0$). We denote the field fluctuations by $\delta\phi^i = \phi^i - \langle \phi^i \rangle$. In normal coordinates k^{kl}_{ij} used below is actually $k^{kl}_{ij} = R^{kl}_{ij}$.

From the eqs of motion for F^i, ϕ^i, after taking the vev's, then

$$k^j_i \, \langle F^\dagger_j \rangle + f_i = 0, \qquad k^j_{im} \, \langle F^i \rangle \langle F^\dagger_j \rangle + f_{km} \langle F^m \rangle = 0 \tag{7}$$

We denote by f_i, f_{ik}, f_{ijk}, the values of corresponding, field dependent W_i, W_{ik}, W_{ijk}, evaluated on the ground state, so

$$f_i = W_i(\langle \phi^m \rangle), \quad f_{ij} = W_{ij}(\langle \phi^m \rangle), \quad f_{ijk} = W_{ijk}(\langle \phi^m \rangle), \quad f^i = W^i(\langle \phi^m \rangle), \text{ etc.} \tag{8}$$

Eq.(7) then becomes

$$\langle F^\dagger_j \rangle = -f_j, \qquad f_{km}\langle F^m \rangle = 0. \tag{9}$$

To break supersymmetry, a non-vanishing vev of an auxiliary field is needed, which requires $\det f_{ij} = 0$. The goldstino mass matrix is $(M_F)_{ij} = W_{ij} - \Gamma^m_{ij} W_m$ evaluated on the ground state, giving $(M_F)_{ij} = f_{ij}$ in normal coordinates. A consequence of the last eq in (9) is that the goldstino eigenvector (normalised to unity) is

$$\tilde{\psi}^1 = -\frac{\langle F^\dagger_m \rangle \psi^m}{\left[\langle F^\dagger_i \rangle \langle F^i \rangle \right]^{1/2}} = \frac{f_m \psi^m}{\left[f_i f^i \right]^{1/2}}, \qquad m_{\tilde{\psi}^1} = 0. \tag{10}$$

Further, regarding the scalar sector, the mass matrix has the form

$$M^2_b = \begin{bmatrix} (V)^k_l & (V)_{kl} \\ (V)^{kl} & (V)^l_k \end{bmatrix} = \begin{bmatrix} f^{ik} f_{il} - k^{jk}_{il} \, f^i f_j & f_{jkl} f^j \\ f^{jkl} f_j & f^{il} f_{ik} - k^{jl}_{ik} \, f^i f_j \end{bmatrix}, \tag{11}$$

where $V^k_l = \partial^2 V/(\partial\phi^l \partial\phi^\dagger_k)$, $V_{kl} = \partial^2 V/(\partial\phi^l \partial\phi^k)$, etc, is evaluated on the ground state.

The two real components of the complex sgoldstino are mass degenerate only if in (11) the off-diagonal (holomorphic or anti-holomorphic) blocks vanish. For simplicity we assume that this is indeed the case. This restricts the generality of our superpotential by the condition

$$f_{ijk} \, f^k = 0, \tag{12}$$

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that we assume to be valid in this paper[1]. In this case, the block $(V)^k{}_l$ determines the mass spectrum and eigenstates. The mass of (complex) sgoldstino obtained from this block must involve Kahler terms (their derivatives), it cannot acquire corrections from f_{ij} and it must be proportional to SUSY breaking, thus it depends on f_i, f^i. The only possibility in normal coordinates is to contract the only non-trivial, non-vanishing tensor $k^{ij}_{kl} = R^{ij}_{kl}$ with f_i, f^i and ensure the correct mass dimension and sign. The result for this (mass)^2 is given in the equation below and a discussion can be found in [8]. Finally since SUSY is broken spontaneously, the sgoldstino mass eigenvector is expected to have a form similar to that of goldstino itself in eq.(10). Indeed, in the limit of ignoring the Kahler part of $(V)^k{}_l$ which is sub-dominant, of order $\mathcal{O}(1/\Lambda^2)$, the sgoldstino is the (massless) eigenvector of $f^{ik} f_{il}$ matrix, and has the form:

$$\tilde{\phi}^1 = \frac{f_m \, \delta\phi^m}{[f^i \, f_i]^{1/2}} + \mathcal{O}(1/\Lambda^2), \qquad m^2_{\tilde{\phi}^1} = -\frac{k^{ij}_{kl} \, f_i \, f_j \, f^k \, f^l}{f_m \, f^m} \tag{13}$$

where $\delta\phi^m = \phi^m - \langle \phi^m \rangle$ is the field fluctuation about the ground state. The mass of sgoldstino $\tilde{\phi}^1$ comes from D-terms, which are $\mathcal{O}(1/\Lambda^2)$.

Spontaneous SUSY breaking suggests the auxiliary of goldstino superfield should have a similar structure:

$$\tilde{F}^1 = \frac{f_m \, F^m}{[f^i \, f_i]^{1/2}} \tag{14}$$

This is verified onshell, when $F^\dagger_i = -W_i + \mathcal{O}(1/\Lambda^2)$ is expanded about the ground state to linear order fluctuations $F^\dagger_i = -f_i - f_{im} \, \delta\phi^m + \mathcal{O}(1/\Lambda^2)$. One then finds from $(14)^2$

$$\tilde{F}^1 = \frac{f_m \, (-f^m)}{[f^i \, f_i]^{1/2}} + \mathcal{O}(1/\Lambda^2) \tag{15}$$

For illustration let us now consider in detail the case of only two fields present in Lagrangian (2), and also present the expression of the second mass eigenvector. For the fermions, the mass eigenvectors (normalised to unity) are given below, with $\tilde{\psi}^1$ the goldstino field:

$$\tilde{\psi}^1 = \frac{1}{[f_i \, f^i]^{1/2}} \left[f_1 \, \psi^1 + f_2 \, \psi^2 \right], \qquad m^2_{\tilde{\psi}^1} = 0.$$

$$\tilde{\psi}^2 = \frac{1}{[f_i \, f^i]^{1/2}} \left[-(f_1/\rho) \, \psi^1 + f_2 \, \rho \, \psi^2 \right], \qquad m^2_{\tilde{\psi}^2} = f^{ij} \, f_{ij}, \quad \rho = \frac{|f_1|}{|f_2|}. \tag{16}$$

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[1] An example when such condition is respected is for a superpotential of the type $W = f_1 \Phi^1 + \lambda/6 \, (\Phi^2)^3$, with Φ^1 breaking SUSY and Φ^2 a matter field. This example will be considered later.

[2] Also at minimum, V should be just $|\langle \tilde{F}^1 \rangle|^2$ which is respected.

For the scalars sector, we find after some algebra the mass eigenstates[3] of $(M_b^2) = V_l^k$:

$$
\tilde{\phi}^1 = \frac{1}{[f_i\,f^i]^{1/2}} \left[\, f_1\,(1 + \xi\,\tilde{k}_{11})\,\delta\phi^1 + f_2\,(1 + \xi\,\tilde{k}_{12})\,\delta\phi^2 \right],
$$

$$
\tilde{\phi}^2 = \frac{1}{[f_i\,f^i]^{1/2}} \left[\, -(f_1/\rho)\,(1 + \xi\,\tilde{k}_{21})\,\delta\phi^1 + f_2\,\rho\,(1 + \xi\,\tilde{k}_{22})\,\delta\phi^2 \right], \tag{17}
$$

with the notation

$$
\begin{aligned}
\tilde{k}_{11} &= k_{kl}^{ij}\,\rho_i^k\,f_i f_j f^k f^l, & \tilde{k}_{12} &= k_{kl}^{ij}\,\nu_i^k\,f_i f_j f^k f^l, & \xi &\equiv [2(f_k\,f^k)\,(f^{ij}\,f_{ij})]^{-1} \\
\tilde{k}_{21} &= k_{kl}^{ij}\,\sigma_i^k\,f_i f_j f^k f^l, & \tilde{k}_{22} &= k_{kl}^{ij}\,\delta_i^k\,f_i f_j f^k f^l,
\end{aligned} \tag{18}
$$

where

$$
\begin{aligned}
\rho_1^1 &= 2/\rho^2, & \rho_1^2 &= 1 + \rho^2 + 2/\rho^2, & \rho_2^1 &= -3 - \rho^2, & \rho_2^2 &= -2, \\
\nu_1^1 &= -2, & \nu_1^2 &= -(1 - \rho^2), & \nu_2^1 &= -(1 - \rho^2), & \nu_2^2 &= 2\,\rho^2, \\
\sigma_1^1 &= -2, & \sigma_1^2 &= 1 + 1/\rho^2 + 2\rho^2, & \sigma_2^1 &= -3 - 1/\rho^2, & \sigma_2^2 &= 2\,\rho^2, \\
\delta_1^1 &= 2/\rho^2, & \delta_1^2 &= -(1 - 1/\rho^2), & \delta_2^1 &= -(1 - 1/\rho^2), & \delta_2^2 &= -2.
\end{aligned} \tag{19}
$$

In our normal coordinates $k_{ij}^{kl} = R_{ij}^{kl}$. We also find the masses:

$$
m_{\tilde{\phi}^1}^2 = -\frac{k_{kl}^{ij}\,f_i f_j f^k f^l}{f_m f^m}, \qquad m_{\tilde{\phi}^2}^2 = f^{ij} f_{ij} - k_{ik}^{jk}\,f^i f_j + \frac{k_{kl}^{ij}\,f_i f_j f^k f^l}{f^m f_m}. \tag{20}
$$

We identify $\tilde{\phi}^1$ of (17) as the sgoldstino, since its mass should not receive corrections from f_{ij}, in the limit of ignoring the curvature tensor corrections in (17), and it has a form similar to that of goldstino eigenstate (10), (16). Regarding the auxiliary fields, one can show that $\tilde{F}^2 = O(1/\Lambda^2)$ and that $\tilde{F}^1$ is that in (15).

We conclude that the goldstino superfield has the onshell SUSY form

$$
\tilde{\Phi}^1\big|_{\text{on-shell}} = \frac{f_k\,\delta\Phi^k}{[f^i f_i]^{1/2}}\big|_{\text{on-shell}} + \mathcal{O}(1/\Lambda^2)
$$

$$
\delta\Phi^k\big|_{\text{on-shell}} \equiv \delta\phi^k + \sqrt{2}\,\theta\,\psi^k + \theta\theta\,(-f^k). \tag{21}
$$

where we used that $k_{kl}^{ij} = \mathcal{O}(1/\Lambda^2)$ and that auxiliary fields are on-shell.

Eqs.(17) to (21) are valid under the assumption that corrections suppressed by powers of Λ are sub-leading to the superpotential SUSY corrections, proportional to f_{ij}, see also

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[3] eqs.(16) and (17) are multiplied in the rhs by $|f_2|/f_2$ which is set to unity by phase rescaling Φ_2.

(11). Let us introduce a parameter ζ equal to the ratio of the Kahler curvature tensor contracted by the SUSY breaking scale(s) f_i to the SUSY "mass term" (f_{ij}):

$$\zeta = \xi\,\tilde{k}_{ij} \sim \frac{k_{ij}^{kl}\,f^i f^j f_k f_l}{(f^p f_p)(f_{mn}f^{mn})} \sim \frac{m_{\text{sgoldstino}}^2}{f_{mn}f^{mn}} \leq 1. \tag{22}$$

If $\zeta \leq 1$ the results of this section such as (17) and (21) are valid and terms suppressed by high powers of Λ can be neglected, as we actually did. For $\zeta \sim 1$ the eigenvectors have a more complicated form (easily obtained) and is not presented here. The limit $\zeta \gg 1$ corresponds to decoupling a massive sgoldstino and is discussed in Section 2.4

We shall compare eq.(21) to the chiral superfield X that breaks superconformal symmetry, conjectured in [1] to be equal, in the infrared limit to the goldstino superfield $\tilde{\Phi}^1$.

2.2 The chiral superfield X and its low-energy limit.

Let us explore the properties of the superconformal symmetry breaking chiral superfield X and examine its relation to the goldstino superfield found earlier. The definition of X is

$$\overline{D}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = D_\alpha X, \qquad X \equiv (\phi_X, \psi_X, F_X) \tag{23}$$

where $\mathcal{J}$ is the Ferrara-Zumino current [2]. For a review of this topic, see for example section 2.1 in [1]. ψ_X is related to the supersymmetry current and F_X to the energy-momentum tensor. For the general, non-normalizable action in (2), this equation has a solution [9]

$$X = 4\,W - \frac{1}{3}\,\overline{D}^2\,K - \frac{1}{2}\,\overline{D}^2 Y^\dagger(\Phi^\dagger) \tag{24}$$

We find the component fields of X to be (ignoring the improvement term $\overline{D}^2 Y^\dagger(\Phi^\dagger)$):

$$
\begin{aligned}
\phi_X &= 4\,W(\phi^i) + \frac{4}{3}\left[K^j\,F_j^\dagger - \frac{1}{2}K^{ij}\,\overline{\psi}_i\overline{\psi}_j\right] \\
\psi_X &= \psi^k\,\frac{\partial\phi_X}{\partial\phi^k} - \frac{4\,i}{3}\sigma^\mu\left(K^j\,\partial_\mu\overline{\psi}_j + K^{ij}\,\overline{\psi}_j\partial_\mu\phi_i^\dagger\right) \\
F_X &= F^i\,\frac{\partial\phi_X}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\,\frac{\partial^2\phi_X}{\partial\phi^i\partial\phi^j} + \frac{4}{3}\left\{K_i^j\left[\partial_\mu\phi^i\partial^\mu\phi_j^\dagger + \frac{i}{2}\left(\psi^i\sigma^\mu D_\mu\overline{\psi}_j - D_\mu\psi^i\,\sigma^\mu\overline{\psi}_j\right)\right]\right. \\
&\quad \left. - \partial_\mu\left(K^j\,\partial_\mu\phi_j^\dagger - \frac{i}{2}K_i^j\psi^i\sigma^\mu\overline{\psi}_j\right)\right\}
\end{aligned}
\tag{25}
$$

In these relations all derivatives are scalar-fields dependent quantities. As a side-remark, one also notices that the integer powers $n \geq 1$ of these components have a nice compact structure:

$$\phi_{X^n} = (\phi_X)^n, \qquad (n \geq 1)$$

$$\psi_{X^n} = n\,(\phi_X)^{n-1}\,\psi_X = \psi^j\,\frac{\partial \phi_{X^n}}{\partial \phi^j} + \mathcal{O}(\partial_\mu)$$

$$F_{X^n} = n\,(\phi_X)^{n-2}\left[\phi_X\,F_X - \frac{n-1}{2}\,\psi_X\psi_X\right] = F^j\,\frac{\partial \phi_{X^n}}{\partial \phi^j} - \frac{1}{2}\psi^i\psi^j\,\frac{\partial^2 \phi_{X^n}}{\partial \phi^i \partial \phi^j} + \mathcal{O}(\partial_\mu),\ (26)$$

where the terms $\mathcal{O}(\partial_\mu)$ vanish in the infrared limit of zero momenta.

Notice that in the leading (zero-th) order in $1/\Lambda$, the only dependence of these components on the Kahler comes through ϕ_X via its term $K^j\,F_j^\dagger$, with additional contributions, fermionic dependent being[4] $\mathcal{O}(1/\Lambda)$.

From (25) we expand X about the ground state and denote $w = W(\langle \phi^k \rangle)$. Keeping linear fluctuations in fields, one obtains from eq.(25) that

$$\phi_X = 4\,w + \frac{8}{3}\,f_j\,\delta\phi^j + \mathcal{O}(1/\Lambda),$$

$$\psi_X = \frac{8}{3}\,f_k\,\psi^k + \mathcal{O}(1/\Lambda),$$

$$F_X = \frac{8}{3}\,f_k\,(-f^k) - 4\,f^k\,f_{km}\,\delta\phi^m - \frac{4}{3}\,f^k\,\delta F_k^\dagger + \frac{8}{3}\,f_k\,\delta F^k + \mathcal{O}(1/\Lambda). \qquad (27)$$

Up to a constant we can write, using eq.(21), that onshell-SUSY:

$$X\big|_{\text{on-shell}} = \frac{8}{3}\,f_k\left[\delta\phi^k + \sqrt{2}\,\theta\,\psi^k + \theta\theta\,(-f^k)\right] + \mathcal{O}(1/\Lambda)$$

$$= \frac{8}{3}\,f_k\,\delta\Phi^k\big|_{\text{on-shell}} + \mathcal{O}(1/\Lambda) \qquad (28)$$

Comparing this result against that for the goldstino superfield of (21), one has

$$X\big|_{\text{on-shell}} = \frac{8}{3}\,\sqrt{f_i f^i}\,\tilde{\Phi}^1\big|_{\text{on-shell}} + \mathcal{O}(1/\Lambda). \qquad (29)$$

Note that the X field goes to the (onshell) goldstino field in the limit of vanishing momentum and in addition $\Lambda \to \infty$ when higher dimensional terms in the Kahler potential decouple. This clarifies the relation between the goldstino and the superconformal symmetry breaking superfields for general K and W, in the presence of more sources of SUSY breaking, and is one of the results of this work. All directions of supersymmetry breaking contribute to the relation between these two superfields.

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[4] The bracket in F_{X^n} is SUSY invariant for $n = 2$, and then $F_{X^2} = 0$ is invariant.

2.3 Further properties of the field X.

Let us compute the onshell form of X by eliminating the auxiliary fields F^k in (25) and then examine under what conditions X^2 could vanish. The results below are valid up to $\mathcal{O}(\partial_\mu)$, where all terms K^j, W_k... etc are actually scalar-fields dependent. One has

$$
\begin{aligned}
\phi_X &= \sigma + \sigma^{mn}\,\overline{\psi}_m \overline{\psi}_n, \\
\psi_X &= \frac{8}{3}\,\psi^k\,W_k, \\
F_X &= \beta + \beta_{mn}\,\psi^m \psi^n + \beta^{kl}_{mn}\,(\psi^m \psi^n)\,(\overline{\psi}_k \overline{\psi}_l),
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
\sigma &= 4\,W - (4/3)\,K^l\,(K^{-1})^k_l\,W_k, \\
\sigma^{mn} &= (2/3)\left[K^l\,\Gamma^{mn}_l - K^{mn}\right] = \mathcal{O}(1/\Lambda), \\
\beta &= -(8/3)\,W_m\,(K^{-1})^m_k\,W^k, \\
\beta_{mn} &= 2\left[W_k\,\Gamma^k_{mn} - W_{mn}\right] = -2\,W_{mn} + \mathcal{O}(1/\Lambda), \\
\beta^{kl}_{mn} &= (1/3)\left(K^{kl}_{mn} - K^j_{mn}\,\Gamma^{kl}_i\right) \equiv (1/3)\,R^{kl}_{mn} = \mathcal{O}(1/\Lambda^2).
\end{aligned}
\tag{31}
$$

Here we made explicit the terms which are suppressed by powers of the cutoff scale.

In [1, 3] it was used that the constraint $X^2 = 0$ projects out the sgoldstino field[5]. In a strict sense this constraint is valid only in the limit of an infinite sgoldstino mass. So the problem is that one has an expansion in $1/\Lambda$ of the initial Lagrangian which can conflict with an expansion in the inverse sgoldstino mass, $1/m^2_{\tilde\phi_1} \sim 1/(f_i^2/\Lambda^2) = \Lambda^2/f_i^2$, that decouples the sgoldstino. The effective Kahler terms must give a mass to sgoldstino (which would otherwise be massless at tree level in spontaneous Susy breaking), and must simultaneously be large enough for the sgoldstino to decouple at low energy. The two expansions may have only a very small overlap region of simultaneous convergence.

To have $X^2 = 0$ it is necessary and sufficient to have $F_{X^2} = 0$. This can be checked directly. If for example $F_{X^2} = 0$ then one immediately shows that $\phi_X \sim \psi_X \psi_X$ so $X^2 = 0$. Let us compute the value of F_{X^2} in general, using (30). One has

$$
\begin{aligned}
F_{X^2} &= \alpha + \alpha_{mn}\,(\psi^m \psi^n) + \lambda^{mn}\,(\overline{\psi}_m \overline{\psi}_n) + \nu^{kl}_{mn}\,(\psi^m \psi^n)(\overline{\psi}_k \overline{\psi}_l) + \xi_{mn}\,(\psi^m \psi^n)(\overline{\psi}_1 \overline{\psi}_1)(\overline{\psi}_2 \overline{\psi}_2) \\
&= \alpha + \alpha_{mn}\,(\psi^m \psi^n) + \mathcal{O}(1/\Lambda)
\end{aligned}
\tag{32}
$$

with

$$
\alpha = 2\sigma\beta, \qquad\qquad \alpha_{mn} = 2\sigma\beta_{mn} - \frac{64}{9}W_m W_n,
\tag{33}
$$

---

[5]Additional constraints for matter superfields can be used to decouple superpartners at low energy.

while the remaining coefficients are suppressed, of order $\mathcal{O}(1/\Lambda)$ or higher[6] and vanish at large Λ. In this limit only, expanding (32) about the ground state (or using (27)) we find

$$F_{X^2} = -\frac{64}{9}\left(2\,f_k f^k f_l\,\delta\phi^l + f_k f_l\,\psi^k \psi^l\right) + \mathcal{O}(1/\Lambda) \tag{34}$$

up to a constant ($\propto w$). To see if F_{X^2} and thus X^2 vanish (hereafter this is considered up to $\mathcal{O}(1/\Lambda)$ terms) after decoupling massive scalar fields, one should integrate out $\delta\phi^k$, $k = 1, 2...$, via the eqs of motion. With $\delta\phi^k$ expressed in terms of the light fermionic and other scalar degrees of freedom, one checks in this way if $X^2 = 0$, without the need of computing the mass eigenstates[7]. In general, upon integrating out the sgoldstino and additional massive scalars, X^2 necessarily contains terms suppressed by the sgoldstino mass. In most cases X^2 does not vanish anymore if this mass is *finite*, except in specific cases, due to additional simplifying assumptions (symmetries, etc) for the terms (e.g. k_{mn}^{ij}) of the Lagrangian. In these cases the convergence problem mentioned earlier is not an issue. We discuss such a case in the next two sections.

2.4 Decoupling all scalar fields, for vanishing SUSY mass terms.

Let us consider the special case of a vanishing SUSY term, i.e. $f_{ij} = 0$ or assume it is much smaller than the Kahler terms in the mass matrix of eq.(11). We consider only two fields present in the Lagrangian (2), both of which can contribute to the SUSY breaking. This can be generalised to more fields. For $f_{ij} = 0$ we have two massless fermions. As a result, both scalar fields, which are massive (via Kahler terms), can be integrated out and expressed in terms of these massless fermions, without special restrictions for scales present. We shall do this and then examine under what conditions X^2 vanishes after decoupling.

Regarding the relation of X to the goldstino superfield, in this case it is difficult to define the latter, as both fields contribute to SUSY breaking and they can also mix. In previous sections, see eq.(22), (29) the superpotential (f_{ij}) terms were dominant and Kahler terms were a small correction ($\propto 1/\Lambda^2$) to the scalars mass matrix (11), while here the situation is reversed. Eq.(11) with vanishing off-diagonal blocks and vanishing $f^{ik} f_{il}$ gives a mass matrix $M_b^2 = (V)_l^k = -k_{il}^{jk} f^i f_j$ in basis $\delta\phi^{1,2}$ with eigenvalues

$$m_{\tilde\phi^{1,2}}^2 = \frac{1}{2}\Big[-k_{mk}^{mj}\,f^k f_j \pm \sqrt{\Delta}\Big] + \mathcal{O}(f^{ij}f_{ij}), \quad \Delta = (k_{mk}^{mj}\,f^k f_j)^2 - 4\det(k_{mk}^{pj}\,f^k f_j) \tag{35}$$

where all indices take values 1 and 2, and the determinant is over the free indices. This is the counterpart to the result in (20). The eigenvectors can also be found[8]. How do we

[6]The expressions of these terms are $\lambda^{mn} = 2\,\beta\,\sigma^{mn} = \mathcal{O}(1/\Lambda)$, $\nu_{mn}^{kl} = 2\left[\sigma\,\beta_{mn}^{kl} + \beta_{mn}\sigma^{kl}\right] = \mathcal{O}(1/\Lambda)$, $\xi_{mn} = 2\left[\sigma^{11}\,\beta_{mn}^{22} + \sigma^{22}\,\beta_{mn}^{11} - 2\sigma^{12}\,\beta_{mn}^{12} = \mathcal{O}(1/\Lambda)\right]$.

[7]For one field breaking SUSY, $f^1 \neq 0$, $(f^{j\neq1}=0)$ and if $w = 0$, $F_{X^2} = 0$ if $\delta\phi^1 = \psi^1\psi^1/(-2\,f^1)$ see [1].

[8]They are $\tilde\phi^{1,2} = (1/|\tilde\phi^{1,2}|)\{(2k_{1j}^{2k} f_k f^j)^{-1}\big[(k_{1m}^{1n} - k_{2m}^{2n})f^m f_n \pm \sqrt{\Delta}\big]\delta\phi^1 + \delta\phi^2\} + \mathcal{O}(f^{ij} f_{ij})$ ($|\tilde\phi^{1,2}|$: norm).

identify the sgoldstino? The term $f_{ij}f^{jk}$ which defined in (11) the leading contribution to the mass matrix and eigenvectors, is vanishing, so it cannot be used. One can identify the sgoldstino from a transformation that ensures that only one linear combination of auxiliary fields breaks supersymmetry. The scalar in the same supermultiplet is then the sgoldstino; further, if no mixing is induced by Kahler curvature terms (this mixing is controlled by k_{mn}^{ij} and is therefore UV and model-dependent) then this state is also a mass eigenstate. To this end define new superfields

$$
\begin{aligned}
\tilde{\Phi}^1 &= \frac{1}{[f_k f^k]^{1/2}} \left(f_1\, \delta\Phi^1 + f_2\, \delta\Phi^2 \right), \\
\tilde{\Phi}^2 &= \frac{1}{[f_k f^k]^{1/2}} \left(-(f_1/\rho)\, \delta\Phi^1 + f_2\, \rho\, \delta\Phi^2 \right), \qquad \rho = |f_1|/|f_2|.
\end{aligned}
\tag{36}
$$

where $\delta\Phi^j = (\delta\phi^j, \psi^j, F^j)$. $\tilde{\Phi}^1$ is inferred from the auxiliary fields combination and $\tilde{\Phi}^2$ was determined by unitarity arguments. One can apply this transformation to the original Lagrangian, then if scalar components are not mixing, $\tilde{\phi}^1$ is also a mass eigenstate[9].

Let us now discuss the decoupling of the scalars and check under what conditions X^2 can vanish, without demanding an infinite sgoldstino mass (which would bring convergence problems). We integrate the scalars, so in the low energy they are combinations of the light/massless fermions. To this purpose, we do not need to identify the sgoldstino. From (5), the eq of motion of scalar field $\phi_l^\dagger$, at zero-momentum, is

$$
W^{kl}(K^{-1})_k^i W_i + W^k(K^{-1})_k^{il} W_i + \frac{1}{2}\left(W^{ijl} - \partial^l(\Gamma_m^{ij} W^m) \right)\overline{\psi}_i\overline{\psi}_j - \frac{1}{2}\partial^l \Gamma_{ij}^m\, W_m \psi^i \psi^j = 0. \tag{37}
$$

We expand this about the ground state, in normal coordinates and use our simplifying assumptions

$$
f_{ij} = 0, \qquad f^{ijl} f_l = 0, \qquad \text{and} \qquad f^{ijlm} = 0. \tag{38}
$$

The result is

$$
k_{kj}^{il}\, \delta\phi^j\; f^k f_i + \frac{1}{2} k_{ij}^{lm}\, f_m\, \psi^i \psi^j - \frac{1}{2} f^{ijl}\, \overline{\psi}_i\overline{\psi}_j + \mathcal{O}(1/\Lambda^3) = 0, \qquad i,j,k,l,m = 1,2. \tag{39}
$$

Taking $l = 1, 2$, we solve this system for $\delta\phi^{1,2}$ to find

$$
\begin{aligned}
\delta\phi^1 &= \frac{1}{2\det(k_{lm}^{kn} f_n f^m)} \left[A_{ij}\, \psi^i \psi^j + B^{ij}\, \overline{\psi}_i\overline{\psi}_j \right] + \mathcal{O}(1/\Lambda) \\
\delta\phi^2 &= \frac{1}{2\det(k_{lm}^{kn} f_n f^m)} \left[C_{ij}\, \psi^i \psi^j + D^{ij}\, \overline{\psi}_i\overline{\psi}_j) \right] + \mathcal{O}(1/\Lambda)
\end{aligned}
\tag{40}
$$

---

[9] This requires a diagonal mass matrix in $\delta\phi^{1,2}$ initial basis, i.e. $k_{2m}^{1k} f_k f^m = k_{1m}^{2k} f_k f^m = 0$ (for a vanishing f_2 this means $k_{12}^{11} = k_{11}^{12} = 0$). In this case the masses are $m_{\tilde{\phi}^1}^2 = -k_{1n}^{1m} f_m f^n$, $m_{\tilde{\phi}^2}^2 = -k_{2n}^{2m} f_m f^n$.

with

$$\begin{aligned}
A_{ij} &= \left(k_{ij}^{2p}\,k_{2s}^{1r} - k_{ij}^{1p}\,k_{2s}^{2r}\right) f^r f_s f_p, \quad B^{ij} = -f^{ij2}\,k_{2s}^{mr}\,f^s f_m f_r (f_1)^{-1}, \\
C_{ij} &= \left(k_{ij}^{1p}\,k_{1s}^{2r} - k_{ij}^{2p}\,k_{1s}^{1r}\right) f^r f_s f_p, \quad D^{ij} = -f^{ij1}\,k_{1s}^{mr}\,f^s f_m f_r (f_2)^{-1}.
\end{aligned} \tag{41}$$

The fields are suppressed by the mass of sgoldstino since the determinant in the denominator is a product of scalar masses, see (35); but due to interactions (f^{ijl}), by counting the mass dimensions, the expansion can also be regarded as proportional to Λ^2/f_i^2 ! Indeed:

$$\delta\phi^1 \propto \frac{1}{m_{\text{sgoldstino}}^2}\left[A_{ij}\,\psi^i\psi^j + B_{ij}\,\overline{\psi}_i\overline{\psi}_j\right], \qquad \text{(similar for } \delta\phi^2\text{)}. \tag{42}$$

This is the sgoldstino decoupling limit, opposite to that considered in eq.(22).

To check if X^2 vanishes for finite sgoldstino/scalars masses, we use the result of (34). From (40) one finds that this happens if

$$(f_k f^k)\left[f_1\,A_{ij} + f_2\,C_{ij}\right] + \det(\tilde{k}_n^m)\,f_i f_j = 0, \qquad f_1\,B^{ij} + f_2\,D^{ij} = 0 \tag{43}$$

with $\tilde{k}_n^m \equiv k_{ns}^{mr}\,f_r f^s$. The last two equations can be re-written as

$$\begin{aligned}
f_p\left[k_{ij}^{2p}\,(f_1\,\tilde{k}_2^1 - f_2\,\tilde{k}_1^1) - k_{ij}^{1p}\,(f_1\,\tilde{k}_2^2 - f_2\,\tilde{k}_1^2)\right] + \det(\tilde{k}_n^m)\,\frac{f_i f_j}{f_k f^k} &= 0 \\
k_{ls}^{mr}\,f^{ijl}\,f^s f_m f_r &= 0,
\end{aligned} \tag{44}$$

where i, j are fixed to any value, 1,2. If these relations are respected one has in the model $X^2 = 0$ for a finite sgoldstino mass and trilinear interactions in the superpotential. These relations ultimately imply some constraints for the curvature tensor and thus for the UV regime. The first relation in (44) simplifies further in specific cases, for example if $\tilde{\phi}^1$ of (36) is also a mass eigenstate which happens for $\tilde{k}_2^1 = \tilde{k}_1^2 = 0$. Conditions (44) can be generalised to more fields and should be verified in those applications in which the constraint $X^2 = 0$ was used. These conditions would also be recovered with our definition of the goldstino superfield in (36). With this definition, the above relations are obtained by demanding (onshell) $(\tilde{\Phi}^1)^2 = 0$, or equivalently $F_{(\tilde{\Phi}^1)^2} = 2\tilde{F}^1\,\tilde{\phi}^1 - \tilde{\psi}^1\tilde{\psi}^1 = 0$.

To illustrate some implications, let us take in eq.(40) the limit of only one field breaking supersymmetry, i.e. assume $f_2 = 0$. One finds

$$\begin{aligned}
\delta\phi^1 &= -\frac{\psi^1\psi^1}{2\,f^1} + \frac{\det(k_{2j}^{1i})}{\det(k_{1n}^{1m})}\frac{\psi^2\psi^2}{2\,f^1} - \frac{k_{21}^{11}\,f^{ij2}}{\det(k_{1n}^{1m})}\frac{\overline{\psi}_i\overline{\psi}_j}{2\,|f_1|^2} + \mathcal{O}(1/\Lambda) \\
\delta\phi^2 &= -\frac{\psi^1\psi^2}{f^1} + \frac{k_{11}^{12}\,k_{22}^{11} - k_{11}^{11}\,k_{22}^{12}}{\det(k_{1n}^{1m})}\frac{\psi^2\psi^2}{2\,f^1} + \frac{k_{11}^{11}\,f^{ij2}}{\det(k_{1n}^{1m})}\frac{\overline{\psi}_i\overline{\psi}_j}{2\,|f_1|^2} + \mathcal{O}(1/\Lambda)
\end{aligned} \tag{45}$$

This is the general result for the scalars as functions of the massless fermionic fields, when superpotential interactions are present[10]. This result recovers eqs.(33), (37) in [6] but have additional corrections due to superpotential couplings. The terms proportional to f^{ij2} in both $\delta\phi^{1,2}$ are actually dominant, since they grow like Λ^2, as it can be seen from the mass dimensions of the k^{ij}_{lm}. The other terms, coefficients of $\psi^1\psi^1$ and $\psi^2\psi^2$ are actually independent of Λ, although for $\psi^2\psi^2$ they involve UV details[11].

From (34) one obtains $F_{X^2} \propto (2\,f^1\,\delta\phi^1 + \psi^1\psi^1)$ [1], which we demand to vanish. Using $\delta\phi^1$ of (45) or directly from the two equations in (44), one finds that $X^2 = 0$ if

$$\det(k^{1i}_{2j}) = 0, \qquad \text{and} \qquad f^{ij2}\,k^{11}_{12} = 0. \tag{46}$$

These constraints are a particular case of the general conditions in (44). If the Lagrangian respects these conditions, one can have[12] $X^2 = 0$ in the presence of trilinear interactions, with one field breaking SUSY and finite mass sgoldstino. Finally, let us add that eq.(45) and conditions (46) simplify further if one demands $\tilde{\phi}^1$ of (36) be also a mass eigenstate which only happens under a special, additional UV assumption: $k^{11}_{12} = k^{12}_{11} = 0$. Then condition (46) reduces to $k^{11}_{22} = 0$. This is however a particular case, not considered further.

2.5 Decoupling the sgoldstino in the presence of a light matter field.

There are situations when the sgoldstino is significantly heavier than other scalar (matter) fields and is the first or the sole field to decouple at low energy. If so, under what conditions is $X^2 = 0$? To examine this briefly, consider the case of the previous section, of two fields $\Phi^{1,2}$ in the Lagrangian, with a simple superpotential

$$W = f_1\Phi^1 + \frac{\lambda}{3!}\,(\Phi^2)^3, \tag{47}$$

So Φ^1 breaks supersymmetry and we also assume that its scalar component (sgoldstino) is much heavier than the second scalar (matter) field belonging to Φ^2. One can ensure such mass hierarchy by assuming that $\det(k^{1n}_{1m})$ is small enough, see (35). Although we do not consider here the extreme case when it actually vanishes, in that case one has (if $k^{11}_{11} + k^{12}_{12} < 0$) that $h^{11}_{11}\,k^{12}_{12} - k^{11}_{12}\,k^{12}_{11} \approx 0$, $m^2_{\phi^1} \approx -(k^{11}_{11} + k^{12}_{12})\,f^2_1$, $m^2_{\phi^2} \approx 0$. Let us then integrate out the sgoldstino. Its eq of motion, from Lagrangian (5) or (39), is

$$\delta\phi^1 = -\frac{1}{f_1\,k^{11}_{11}}\left[(1/2)\,k^{11}_{mn}\,\psi^m\psi^n + f_1\,\delta\phi^2\,k^{11}_{12}\right] + \mathcal{O}(1/\Lambda), \tag{48}$$

which is a function of the light scalar and massless fermions. Using (48) one finds

---

[10]As usual, in the normal coordinates used here $k^{mn}_{ij} = R^{mn}_{ij}$.

[11]Similar effects were discussed in [5, 6].

[12]up to $\mathcal{O}(1/\Lambda)$ corrections.

$$F_{X^2} \;=\; \frac{64}{9} \frac{(f_1)^2}{k_{11}^{11}} \left[2\, k_{12}^{11}\, (f_1\, \delta\phi^2 + \psi^1\psi^2) + k_{22}^{11}\, \psi^2\psi^2 \right] + \mathcal{O}(1/\Lambda) \tag{49}$$

For any value of the scalar matter field (ϕ^2), with sgoldstino decoupled at finite mass, one can thus have $F_{X^2} = X^2 = 0$ only if $k_{12}^{11} = 0$, $k_{22}^{11} = 0$ and for a large Λ. These conditions can be compared to those when both scalars are decoupled shown in (46) (with $f^{ij2} \to \lambda$). Therefore the action for which the formalism of [1] applies with $X^2 = 0$, has K given by

$$\begin{aligned} K \;=\;& \Phi_1^\dagger \Phi_1 + \Phi_1^\dagger \Phi_1 + k_{11}^{11}\, (\Phi_1^\dagger \Phi_1)^2 + k_{22}^{22}\, (\Phi_2^\dagger \Phi_2)^2 + \left[\, k_{22}^{21}\, (\Phi_2^\dagger \Phi_2)(\Phi_2 \Phi_1^\dagger) + h.c. \right] \\ &+\; k_{12}^{12}\, (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \mathcal{O}(1/\Lambda^3) \end{aligned} \tag{50}$$

with a nontrivial superpotential as in (47) and a finite sgoldstino mass.

In the Lagrangian obtained after decoupling $\delta\phi^1$ one can now also integrate out $\delta\phi^2$ and obtain a solution for it as in (45) but with the replacement $f^{ij2}\overline{\psi}_i\overline{\psi}_j \to \lambda\overline{\psi}_2\overline{\psi}_2$. This solution, if used in (48), brings $\delta\phi^1$ to the form shown in (45), as expected. With this $\delta\phi^2$ one then easily verifies that F_{X^2} of (49) becomes

$$F_{X^2} = \frac{64}{9} \frac{-(f_1)^2}{\det(k_{1n}^{1m})} \left\{ \det(k_{2j}^{1i})\, \psi^2\psi^2 - \frac{\lambda\, k_{12}^{11}}{f_1}\, \overline{\psi}_2\overline{\psi}_2 \right\} \tag{51}$$

and

$$X^2 = -\frac{F_{X^2}}{f^1} \left\{ \frac{-1}{2\, f^1} \left(\psi^1\psi^1 + \frac{9}{128} \frac{F_{X^2}}{(f_1)^2} \right) + \sqrt{2}\,\theta\,\psi^1 + \theta\theta\,(-f^1) \right\} + \mathcal{O}(1/\Lambda). \tag{52}$$

Therefore F_{X^2} vanishes and so does X^2 provided that $\det(k_{2m}^{1l}) = 0$ and $\lambda\, k_{12}^{11} = 0$, and this recovers the result in eq.(46) when both scalars are decoupled.

Higher powers of X can vanish with weaker restrictions. This is actually expected from the properties of the Grassmann variables. Indeed, one shows that in onshell-SUSY case after decoupling only the sgoldstino ($\delta\phi^1$) then:

$$X^3 \propto \frac{k_{12}^{11}}{k_{11}^{11}}\, f_1 \times (\text{function of } \delta\phi^2,\ \psi^{1,2}). \tag{53}$$

This vanishes for any $\delta\phi^2$ and finite sgoldstino mass, provided that $k_{12}^{11} = 0$ which is a weaker constraint than that found for X^2. Higher powers of X show that $k_{12}^{11} = 0$ is still needed for X^4 to vanish for any light matter field, because in (48) $\delta\phi^2$ is multiplied by k_{12}^{11}. Recall however that k_{12}^{11} vanishes if there is no scalars mixing induced by Kahler curvature terms i.e. if ϕ^1 of (47) is also a mass eigenstate.

3   Conclusions.

In this work we considered the relation of the superconformal symmetry breaking chiral superfield X and the goldstino superfield, in effective models with low scale of SUSY breaking, when transverse gravitino couplings are negligible relative to their longitudinal counterparts of its goldstino component. The models considered have a general Kahler (K) and superpotential (W) with more sources of supersymmetry breaking.

In this case we verified the conjecture that the superfield X becomes the goldstino superfield in the limit of zero-momentum and, in addition, $\Lambda \to \infty$, where Λ is the UV cutoff. This happens when the higher dimensional Kahler terms are sub-leading to the supersymmetric mass terms in the scalar mass matrix. For vanishing SUSY mass terms, but otherwise rather general K and W we also investigated the decoupling of the massive scalars simultaneously or separately. In this case we identified the conditions for which the sgoldstino decoupling condition $X^2 = 0$ is still satisfied in the presence of additional fields, for a finite sgoldstino mass. This is important to ensure that the effective expansion $(\propto 1/\Lambda)$ of the Lagrangian does not conflict with the sgoldstino decoupling limit (of small $\propto 1/m^2_{\text{sgoldstino}} \sim \Lambda^2/f_i^2$ where f_i is the SUSY breaking scale).

One can reverse the arguments and conclude that the use of the constraint $X^2 = 0$, although appealing and apparently UV independent, is of somewhat restricted applicability in the case of general K, W (with light fields present, additional SUSY breaking fields, etc); ultimately it implicitly makes assumptions about UV details, difficult to justify without additional input (symmetry, etc). The situation can improve in models where the UV details are under control, such as in renormalizable models of supersymmetry breaking (O'Raifeartaigh, etc), not considered here (where in the sgoldstino decoupling limit Λ is replaced by an appropriate SUSY mass scale).

What does this mean for model building? When parametrizing SUSY breaking in models like the MSSM one commonly uses a spurion field that is a limit of the goldstino superfield with the dynamics integrated out. The above observation regarding UV assumptions suggests that it may be preferable, when studying the details of a low-scale SUSY breaking case, to couple (offshell!) the goldstino superfield to the MSSM, as a *linear* superfield[13] rather than as a non-linear representation that is a solution of the constraint $X^2 = 0$. One can then eventually decouple the sgoldstino explicitly, via the eqs of motion.

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[13]identified as in the text, in the case of more sources of SUSY breaking.

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